Math 1B Midterm 1 Review

It is common for students to assume **mistakenly** that
the first midterm consists entirely of finding indefinite integrals (using substitution and basic techniques),
and evaluating definite integrals (using the Fundamental Theorem of Calculus Part 2).
This is a particularly common mistake among students who have taken integral calculus before.
In fact, there will be relatively few problems of those types.
The following list of supplemental problems demonstrates the diversity of questions you might expect to see.
They are modeled on assigned homework problems.
(There are significantly more questions in this review than will appear on the actual midterm.
Some of the question types below which do not appear on the midterm may appear on the final exam.)

[H] means there is a hint for the question.

As always, you must show proper calculus-level work and fully simplified answers to earn full credit.

[1] The table gives the velocity of a car (in feet/second) at various times (in seconds). At time $t=1$, the position of the car was 21 feet.

t	0	1	2	3	4	5	6	7	8	9	10	11	12	13
$v(t)$	10	12	11	14	15	18	20	24	22	17	13	8	7	5

[a] Write an expression involving an integral for the position of the car at $t=13$.
[b] Estimate the position of the car at $t=13$ using [a], 3 subintervals and
[i] left endpoints [ii] midpoints [iii] right endpoints

[2][H] Find $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{\pi}{n}\sin\left(\pi+\frac{i\pi}{2n}\right)$ by finding the corresponding definite integral, and evaluating that integral.

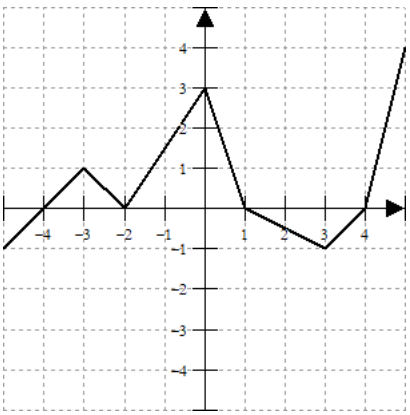
[3] Use the definition of the definite integral, with right endpoints, to evaluate $\int_{-1}^{2}(x^2+3x+2)\,dx$.
DO NOT USE THE FUNDAMENTAL THEOREM OF CALCULUS.

[4][H] Evaluate $\int_{-2}^{8}(x-\sqrt{25-(x-3)^2})\,dx$.
DO NOT USE THE FUNDAMENTAL THEOREM OF CALCULUS. (IT WOULD BE VERY DIFFICULT ANYWAY.)

[5][H] Prove that $\frac{\pi^2}{18}\le\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}x\sin x\,dx\le\frac{\pi^2}{9}$.

[6] Write $\int_{7}^{-5}f(x)\,dx+\int_{-1}^{10}f(x)\,dx-\int_{7}^{10}f(x)\,dx$ using as few integrals as possible.

[7][H] The graph of $f(t)$ is shown on the right. Let $g(x)=\int_{-2}^{x}f(t)\,dt$.



[a] Find $g(3)$ and $g'(3)$.
[b] Find all critical numbers of g.
[c] Find all intervals on which g is decreasing.
[d] Find all local minima of g.
[e] Find all intervals on which g is concave up.
[f] Find all inflection points of g.

[8][H] If $g(x) = \int_{x^2}^{x^3} \ln(1+t^2)\,dt$, find $g''(1)$.

[9][H] Find $f(t)$ and a such that $4 + \int_a^x \frac{1}{f(t)}dt = 2\sqrt{x}$.

[10] If $f(a)$ is the rate (in pounds per year) at which Morgan was gaining weight when he was a years old, what are the meaning and units of $\int_8^{15} f(x)\,dx$?

[11][H] The acceleration of an object at time t (in seconds) is given by $a(t) = 3 - 2t$ (in meters per second^2). The initial velocity of the object is 4 meters per second.

[a] Find the displacement of the object from $t = 1$ to $t = 6$.

[b] Find the distance travelled by (not displacement of) the object from $t = 1$ to $t = 6$.

[12][H] If f is continuous and $\int_{-1}^{5} f(t)\,dt = 7$, find $\int_{-1}^{2} (6 + 4f(2t+1))\,dt$.

[13] Evaluate $\int_{-6}^{6} (x^3\sqrt{2+\cos x} - \sqrt{144-(x+6)^2})\,dx$.

<u>DO NOT USE THE FUNDAMENTAL THEOREM OF CALCULUS. (IT WOULD BE IMPOSSIBLE ANYWAY.)</u>

<u>Don't forget:</u>

THEOREMS

Fundamental Theorem of Calculus (both parts)
Net Change Theorem

<u>And, of course:</u>

You should copy the pages that contain the following questions, cut them up so each question is on a separate slip of paper, throw them all into a pile, pick out questions from the pile randomly, and solve them.

This will help you practice figuring out what technique to use without knowing what section the question came from – an important skill later on when you will learn even more integration techniques.

You don't have to do them all. But do enough of them so that you can recognize what to do in less than 30 seconds.

INTEGRALS

5.3	19-44
5.4	5-46
5.5	7-73
5.REV	11-42

<u>HINTS:</u>

Not so fast. Have you **really** tried to solve the questions already ?